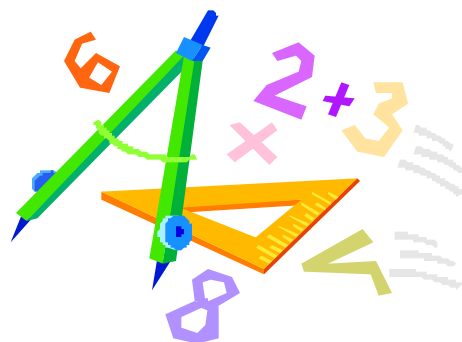


MELROSE PUBLIC SCHOOLS
MATHEMATICS DOCUMENTS



INTRODUCTION

This document was created by Melrose Public Schools staff and administrators after careful consideration of: actual classroom practice, current curriculum documents, the Mass Insight Education Mathematics document of 2002, and the Massachusetts Mathematics Curriculum Frameworks document of November 2000. We support the framework's goal of having all students achieve mathematical competency by participating in a program that emphasizes:

the process of problem solving
effective mathematical communication
the ability to reason and prove an answer
the ability to make connections between math, other subject areas, and life
and the ability to use symbols to represent ideas or actions.

To be competent in the above skills is absolutely essential if a student is to succeed in, and contribute to, the technological society of which he/she is a part.

We also agree that, "Acquiring such competence depends in large part on a **clear, comprehensive, coherent, and developmentally appropriate** set of standards to guide curriculum expectations." ¹ This last statement has, in fact, been the impetus for the creation of this document.

Our district is moving forward with the U.S. Department of Education recommendation to provide ALL students with the opportunity to take Algebra I in Grade 8, and to provide Pre-K through grade 7 students with the foundation they will need to ensure their success in truly understanding algebraic concepts. This will, in turn, allow students the opportunity to take more advanced math and science courses in high school.

Through this course of mathematics study, we hope to instill a sense of curiosity and confidence in our students that will positively affect their performance in math and in life.

Patricia Muxie, Director of Curriculum, Fall 2003

1. p.5, Massachusetts Mathematics Curriculum Framework, November 2000.

MATHEMATICS BENCHMARKS
PRE – KINDERGARTEN

Number Sense and Operations

- Count by ones to at least 10 and up to 20.
- Match quantities up to 10 with numerals.
- Identify position of objects up to 3rd (first, second, third).
- Compare sets of up to at least 5 concrete objects.
- Understand the concepts of whole and half given visual aides.
- Recognize basic operations of addition and subtraction using manipulatives.
- Engage in estimation activities.
- Become familiar with basic math terms such as: add, subtract, equal, none, and equation.

Pattern, Relations, and Algebra

- Identify, sort and classify objects by a single attribute.
- Identify, reproduce, describe, extend and create simple color, rhythmic, and/or shape- repeating patterns.

Geometry

- Name and draw simple two-dimensional shapes with visual cues (circle, square, rhombus, rectangle, triangle, and oval).
- Name three-dimensional shapes (cylinder, prism, sphere).
- Identify position of objects in space using rudimentary language (e.g., beside, next, to, over, under, behind, in front of, on top, over).

Measurement

- Recognize and compare the attributes of length, volume/capacity, weight, area and time using appropriate language (e.g., longer, taller, shorter, same length, heavier, lighter, same weight, holds more, holds less, holds the same amount).
- Make and use estimates of measurements from everyday experiences.
- Use nonstandard units to measure length, area, weight, and capacity.

Data Analysis, Statistics and Probability

- Collect, sort, organize, and draw conclusions about data using concrete objects, pictures, numbers, and graphs.

MATHEMATICS BENCHMARKS KINDERGARTEN

Number Sense

- Count to 20, match quantities up to 10 with numerals and words, and compare sets up to 10 with appropriate language (ex. none, more than, fewer than, one more than).
- Identify positions of objects in sequence up to 5th.
- Use manipulatives / drawings to explore +, and - to 10.
- Estimate the number of objects in a group and verify results.
- Read, write, and order numbers to 12.
- Understand the concepts of whole and half.
- Identify U.S. coins by name.

Geometry

- Name, describe (including number of sides and corners), sort, and draw simple two dimensional shapes.
- Identify position of objects in space using appropriate language (ex. beside, below).

Measurement

- Recognize and compare the attributes of length, weight, capacity and time. Make and use estimates of the above measurements from everyday experiences.
- Use non-standard units to measure length, area, weight, and capacity.

Patterns, Relations and Algebra

- Identify attributes of an object and sort / classify by attributes (ex. a red ball, a square block).
- Identify, describe, produce, reproduce and extend a pattern.
- Count by 5's and 10's to 50.

Data Analysis

- Collect, sort, organize and draw conclusions about data using concrete objects, pictures, numbers, and graphs.

MATHEMATICS BENCHMARKS

GRADE 1

Number Sense

- Demonstrate proper formation of numerals 1 to 100.
- Skip count by 2's, 5's, 10's up to 100.
- Understand and perform an additional and subtraction accurately to 20, including 2-column addition and subtraction with no grouping and solving 3-digit addition with no regrouping.
- Know doubles and halves to and from 20.
- Identify and know the value of a penny, nickel, dime and quarter.
- Trade coins up to 25 cents.
- Use the cent sign accurately.
- Identify and understand ones and tens place value in 2- digit numbers using manipulatives.
- Read and write number words beyond 10.
- Demonstrate reasonable estimating, including estimates of measurement (time, weight, volume, etc.).
- Use ordinal numbers to tenth.
- Identify $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, as parts of a whole and as numbers on a number line.
- Round whole numbers through 100 to the nearest 10.

Geometry

- Recognize, name and compare two and three dimensional shapes.
- Compare length, weight, area, space, and volume of 2 or more objects.
- Use and understand mathematical and geometrical language (ex. rows, columns, length, width, pounds).

Measurement

- Tell time at hour and half- hour intervals on analog and digital clocks.
Measure using standard (inches/feet) and non-standard units.
Identify parts of the day, week, month, and calendar.

Patterns, Relations, and Algebra

- Recognize numbers and patterns on the hundreds chart.
- Identify, reproduce, describe and extend repeating patterns of size, shape, color and number.

Data Analysis

- Use interviews and surveys to gather data, make educational guesses, and draw conclusions.
- Decide and communicate which outcomes are most likely.
- Identify multiple uses of numbers using real-life experiences and objects. (ex. How many are in a group or ordered list? What are the numbers on a watch?)
- Interpret data using bar graphs, pictographs, and Venn diagrams.

MATHEMATICS BENCHMARKS
GRADE 2

Number Sense

- Create and label sets of objects from 0 to 1000.
- Demonstrate an understanding of the concepts of place value – ones, tens, and hundreds to 1,000 using manipulatives (e.g., base ten blocks, number lines).
- Write dictated whole numbers, from 0 to 1,000 with correct place value.
- Identify and represent common fractions ($\frac{_}{_}$ to $\frac{_}{_}$) as parts of wholes, parts of groups, and numbers on the number line.
- Use the terms “numerator” and “denominator” correctly.
- Read, write, compare ($>$, $<$, $=$, odd, even), and order whole numbers to 1,000
- Identify odd and even numbers and determine whether a set of objects has an odd or even number of elements.
- Identify the value of all U.S. coins, and \$1, \$5, \$10, and \$20 bills. Find the value of a collection of coins and bills and different ways to represent an amount of money up to \$5. Use appropriate notation (e.g., \$0.69, \$1.35).
- Explain and demonstrate the relationship between addition and subtraction (fact families).
- Use the appropriate operation (addition or subtraction) to solve single-step problems.
- Know addition facts (addends to 10) and related subtraction facts, and use them to solve problems.
- Demonstrate an understanding of addition and subtraction of any two and three-digit numbers (with and without regrouping).
- Estimate a logical solution to a problem utilizing numbers to 100 and recognize when an estimate is appropriate.
- Explain how an estimate differs from an actual calculation.
- Round whole numbers through 1,000 to the nearest 10 and 100.

Geometry

- Identify, describe, draw, and compare two-dimensional shapes including both polygons (up to six sides) and curved figures, such as circles.
- Demonstrate flips, slides, and turns with manipulatives.
- Identify shapes that have been turned, flipped, slid, and enlarged. Describe the direction, (e.g., left, right, up, down,).
- Identify various lines of symmetry in two dimensional shapes.
- Predict and confirm the results of putting shapes together and taking them apart.

Measurement

- Relate specific months to the seasons.
- Tell and record time to the half hour, and quarter hour using an analog or digital clock, and using a.m. and p.m.

Continued....

Grade 2 Math continued:

- Measure and compare common objects using metric and English units of length measurement.
- Measure to the nearest $\frac{1}{2}$ inch and nearest centimeter.
- Select and correctly use the appropriate measurement tools, (e.g., ruler, balance scale, thermometer).

Patterns / Relationships / Algebra

- Describe and create addition and subtraction number patterns (e.g., 1,4,7,10,...or 25, 23, 21).
- Skip count by twos, fives, and tens to 100, starting at any number.
- Construct and solve open sentences with variables. Recognize that a variable has a value.
- Write number sentences using +,-,<,>, and/or = to represent mathematical relationships in everyday situations.
- Describe functions related to trading, (e.g., 1ft. = 12 in., 4 quarters = \$1.00, 7 days = 1 week).

Data Analysis

- Organize, classify, represent, and interpret data using tallies, charts, tables, bar graphs, pictographs, and Venn diagrams. Formulate inferences.

MATHEMATICS BENCHMARKS
GRADE 3

Number Sense and Operations

- Demonstrate an understanding of the base 10 number system by reading, writing, modeling and interpreting whole numbers to 100,000. Compare and order these numbers.
- Identify and explain common fractions and mixed numbers ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{12}$, $1\frac{1}{2}$) using models and number lines as parts of wholes or parts of a set.
- Give equivalent forms of common fractions less than one whole (e.g., $\frac{2}{4} = \frac{3}{6}$, etc.).
- Use concrete objects and visual models to add and subtract common fractions.
- Identify odds, evens, factors, or multiples of a number and use these as solutions in problems.
- Understand and use the meaning and models of multiplication and division of whole numbers.
- Understand the inverse relationship between \times and \div .
- Select and use appropriate operations ($+$, $-$, \times and \div) to solve problems including money.
- Know multiplication facts to 12×12 and related division facts.
- Add and subtract up to 5-digit numbers using manipulatives, models and conventional algorithms.
- Multiply 3 digits \times 1 digit with regrouping using manipulatives, models and conventional algorithms.
- Divide a 2 digit number by a 1 digit number.
- Round whole numbers through 1,000 to the nearest 100.
- Use the strategies of rounding (e.g., front-end and regrouping) to estimate the results of whole number computations up to 3-digit numbers and amounts of money to \$1,000. Judge the reasonableness of the answer.

Geometry

- Identify, compare and analyze attributes (number of sides, faces, corners, right angles, diagonals, symmetry and congruency) of two and three-dimensional geometric shapes.
- Compare and classify two and three-dimensional shapes, especially triangles and quadrilaterals.
- Use ordered pairs of numbers or letters to locate and identify points.
- Identify/describe lines of symmetry in 2-dimensional shapes.

Measurement

- Understand length, area, weight and volume and select the appropriate type of unit for measuring.
- Carry out simple unit conversions (e.g., hours to minutes, cents to dollars, yards to feet or inches, etc.).

Continued...

Grade 3 Math continued:

- Identify time to the minute on analog and digital clocks using a.m. and p.m.
- Find the perimeter of a polygon measuring and using models and diagrams.
- Find the area of a rectangle.
- Use appropriate metric and standard units and tools (e.g., ruler, graduated cylinder, thermometer) to estimate, measure and solve problems.

Patterns, Relations and Algebra

- Explain geometric and numeric patterns, i.e. 3, 30, 300, 3000.
- Use symbol and letter variables to represent unknowns or quantities that vary in expressions.
- Use, pictures, models, tables, charts, graphs, words, number sentences to interpret/represent mathematical relationships.
 - Solve problems involving proportional relationships (e.g., 4 apples cost \$.80, so one apple costs \$.20).

Data Analysis, Statistics and Probability

- Collect, organize, and display data using observations, measurement, surveys or experiments.
- Match information shown in lists, tables, or graphs with the actual set of data.
- Construct, draw conclusions and make predictions from various representations of data sets (e.g., line plots, bar graphs, etc.).
- Represent the possible outcomes for a simple probability situation.
- Classify outcomes as certain, likely, unlikely or impossible by conducting experiments.

MATHEMATICS BENCHMARKS

GRADE 4

Number Sense and Operations

- Demonstrate an understanding of the base 10 number system by reading, writing and interpreting whole numbers to 1,000,000 and decimals between 0 and 1. Order, compare, round and estimate large numbers through 100,000.
- Demonstrate an understanding of common fractions, including equivalent fractions, mixed numbers, and ordering fractions.
- Identify and generate equivalent forms of common decimals and fractions.
- Accurately compute multiplication facts (12x12, 3 digits x 2 digits) and division facts (3 digits ÷ 2 digits, with and without remainders) using models, manipulatives, and conventional algorithms.
- Select, use, and explain the commutative, associative and identity properties of operations on whole numbers in problem situations (e.g., $37 \times 46 = 46 \times 37$, and $(5 \times 7) \times 2 = 5 (7 \times 2)$).

Geometry

- Describe, compare and analyze attributes of two and three-dimensional figures/shapes. Understand congruency, similarity, slides, flips and turns.
- Identify, define, and construct angles, rays, lines and line segments.
- Apply visual reasoning strategies to fold and combine two and three-dimensional shapes.
- Describe, model, draw, compare, and classify two and three-dimensional shapes (e.g., circles, polygons, especially triangles and quadrilaterals, cubes, spheres, and pyramids).
- Recognize similar figures.
- Identify angles as acute, right, or obtuse.
- Describe and draw intersecting, parallel, and perpendicular lines.

Measurement

- Identify and describe attributes of length, area, volume, time (including elapsed time) and weight and the appropriate unit of measuring. Use and apply information to solve problems.
- Estimate and calculate problems involving perimeter, length, area, volume, weight, time, angle, size and temperature. Use diagrams, models, measuring, and grids as appropriate.

Patterns, Relations and Algebra

- Determine values of variables in simple equations (e.g., $4106 - y = 37$).
- Determine how change in one variable relates to change in a 2nd variable (e.g., input – output table).

Data Analysis, Statistics and Probability

- List and count the number of possible combinations of objects or events from 3 sets.

MATHEMATICS BENCHMARKS

GRADE 5

Number Sense and Operations

- Identify and determine common equivalent fractions, mixed numbers, decimals, and percents.
- Demonstrate an understanding of place value to billions and thousandths.
- Find and position integers, fractions, missed numbers, and decimals, (both positive and negative) on the number line.
- Represent and compare very large and very small numbers.
- Add, subtract, multiply and divide (with double-digit divisors), whole numbers and positive decimals. (Teacher note: Decimals should be in divisor only.)
- Demonstrate an understanding of fractions as a ratio of whole numbers, as parts of unit wholes, as parts of a collection, and as locations on a number line.
- Add, subtract, multiply, and divide positive fractions and mixed numbers. Simplify fractions.
- Estimate results of computations with whole numbers, and with positive fractions, mixed numbers, decimals, and percents. Describe reasonableness of estimates.
- Demonstrate an understanding of inverse relationships in addition and subtraction and use that information to solve problems.

Geometry

- Predict, describe and perform transformations on two-dimensional shapes.
- Identify polygons and 3-D shapes based on their properties, including types of interior angles, perpendicular or parallel sides, and congruence of sides (e.g., squares, rectangles, rhombuses, parallelograms, trapezoids, and isosceles, equilateral, and right triangles).
- Find the distance between 2 points on horizontal or vertical number lines.

Measurement

- Apply the concepts of perimeter and area to the solution of problems. Apply formulas where appropriate.
- Identify, measure, describe, classify, and construct various angles, triangles, and quadrilaterals.
- Find the area of triangles.

Patterns Relations and Algebra

- Analyze and determine the rule for extending patterns.
- Interpret graphs that represent the relationship between two variables in everyday situations.

Data Analysis, Statistics and Probability

- Find the mean, median, mode and range of data.

MATHEMATICS BENCHMARKS GRADE SIX

Number Sense and Operations

- Demonstrate an understanding of positive integer exponents in particular when used in powers of 10 (e.g., $10^_$).
- Perform the operations of addition, subtraction, multiplication, and division of decimals accurately and efficiently.
- Use the number line to model addition and subtraction of integers.
- Convert fractions to simplest form.
- Understand and apply positive integer exponents and percents.
- Perform the operations of addition, subtraction, multiplication, and division of fractions accurately and efficiently.
- Add and subtract integers with the exception of subtracting negative integers.
- Compare integers, fractions, and decimals using the number line to demonstrate their location.
- Utilize the order of operations when simplification of expressions is required.
- Apply number theory concepts, including prime and composite numbers, prime factorization, greatest common factor, least common multiple, and divisibility rules for 2, 3, 4, 5, 6, 9, and 10, to the solution of problems.

Patterns, Relations, and Algebra

- Evaluate expressions with given values for the variables in the expression.
- Solve simple linear equations by utilizing properties of equality using concrete models, tables, graphs, and paper/pencil methods.
- Represent real world situations with concrete models, graphs, and tables.
- Produce and interpret graph results and the relationships between variables.
- Identify and describe relationships between two variables with a constant rate of change.

Geometry

- Identify relationships among points, lines, and planes (e.g., intersecting, parallel, perpendicular).
- Plot points on the coordinate plane in all quadrants.
- Identify types of symmetry including line and rotational.
- Determine if two shapes are congruent by measuring sides or by performing a motion.
- Match 3-D shapes with their 2-D representations.

Measurement

- Find the perimeter and area of plane figures.
- Solve problems involving proportional relationships and units of measurement (e.g., scale model).
- Find the area of parallelograms and develop strategies to find the area of more complex shapes.

Continued...

Grade 6 Math continued:

- Find the circumference and area of a circle.
- Find the volume and surface areas of rectangular prisms.
- Find the sum of angles in simple polygons with and without measuring.

Data Analysis, Statistics, and Probability

- Describe and compare data sets using the concepts of median, mean, mode, maximum and minimum, and range.
- Construct and read **stem-leaf plots**, line graphs, and circle graphs.
- Find the probability of simple events/experiments and use lists or tables to represent outcomes. Analyze outcomes.

MATHEMATICS BENCHMARKS

GRADE 7

Teachers please note: For the 2003-2004 school year, pre-algebra benchmarks (which are listed on the pages that follow Grade 7) should be incorporated into your curriculum.

Number Sense and Operations

- Determine when an estimate rather than an exact answer is appropriate and apply in problem situations.
- Select and use appropriate operations (addition, subtraction, multiplication, division), and positive integer exponents, to solve problems with rational numbers (including negatives).
- Compare and order integers, fractions, and decimals.
- Represent numbers in scientific notation.
- Find the prime factorization of numbers by using the factorization tree.
- Convert percents, fractions, and decimals.
- Find the absolute value of numbers.
- Add, subtract, multiply, and divide decimals.
- Add, subtract, multiply, and divide fractions.
- Simplify powers and use the order of operations.
- Use inverse relationships to solve problems.

Geometry

- Determine the congruency and similarity of figures and apply these relationships to the solution of problems.
- Understand the types of angles formed by intersecting lines or parallel lines cut by a transversal.
- Apply the Pythagorean theorem to obtain missing sides.
- Identify 3-dimensional figures by name.
- Use a straight-edge, compass, or other tools to formulate and test conjectures, and to draw geometric figures.
- Recognize and draw two-dimensional representations of three-dimensional objects (e.g., nets, projections, and perspective drawings).

Measurement

- Find the area and perimeter of polygons.
- Find the area and circumference of circles.
- Convert units of measurement within the metric system or within the same system of measurement.
- Use ratio and proportion to solve for missing angles or sides in similar triangles.

Patterns, Relations, and Algebra

- Evaluate expressions when given the values for variables.
- Solve linear equations in one variable.
- Set up and solve proportions.

Continued...

Grade 7 Math continued:

- Graph a line on a coordinate plane using the table method to obtain coordinates.
- Create and use symbolic expressions and relate them to graphic representations.

Data Analysis, Statistics, and Probability

- Find, describe and interpret measures of central tendency (mean, median, and mode) and spread (range) that represent a set of data. Use these to compare different sets of data.
- Use an outcome tree to determine the possible outcomes of multiple events.
- Find the probability of simple events.

MATHEMATICS BENCHMARKS

PRE-ALGEBRA

Note: During the 2003-2004 school year, these benchmarks will be integrated into the Gr. 7 curriculum.

Number Sense and Operations

- Compare, estimate, and perform operations with integers, fractions, decimals, percents, and mixed numbers.
- Find and compute percentages, rates, and bases.
- Compute percents to determine discounts, percentage increases, commissions, and other uses of percents.
- Work with irrational numbers such as pi and the square root of 2.
- Use proportions to solve word problems including unit rates, scale factors, and rate of change.
- Apply prime factorization to problems when working with number theory.
- Apply the rules of powers and roots to the solution of problems.
- Recognize and distinguish between properties such as commutative associative, distributive; properties of the identify and inverse elements; and the notion of closure of a subset.

Geometry

- Analyze, apply, and explain the relationship between the number of sides and the sums of the interior and exterior angle measures of polygons.
- Demonstrate an understanding of the Pythagorean theorem and its application to the solution of problems.
- Work effectively with transformation, translations, reflections, and rotations.

Measurement

- Select, convert, and use appropriate units of measure.
- Use ratio and proportion (including scale factors) in the solution of problems, including problems involving similar plane figures and indirect measurement.
- Demonstrate an understanding of the concepts and apply formulas and procedures for determining measures, including those of area and perimeter/circumference of parallelograms, trapezoids, and circles. Given the formulas, determine the surface area and volume of rectangular prisms, cylinders, and spheres. Use technology as appropriate.
- Given the formulas, convert from one system of measurement to another. Use technology as appropriate.
- Use models, graphs, and formulas to solve simple problems involving rates (e.g., velocity and density).

Patterns, Relations and Algebra

- Extend, represent, analyze and generalize a variety of patterns with tables, graphs and words, and when possible, symbolic expressions. Include arithmetic and geometric progressions (e.g., compounding).
- Evaluate open expressions for certain values.
- Demonstrate an understanding of the identify $(-x)(-y) = xy$. Use this identity to simplify algebraic expressions (e.g., $(-s)(-x+2) = 2x - 4$). Continued...

Pre-Algebra continued:

- Solve linear equations with two variables.
- Find the slope of lines on a coordinate plane. Apply this concept to the solution of problems.
- Simplify expressions by using the laws of exponents.
- Solve and analyze problems involving proportions.
- Identify the roles of variables within an equation (e.g., Use the slope intercept form of a line to graph).
- Explain and analyze – both quantitatively and qualitatively, using pictures, graphs, charts, or equations – how a change in one variable results in a change in another variable in functional relationships (e.g., $C = d$, $A = r^2$ (A as a function of r), A rectangle = $l w$ (A rectangle as a function of l and w).
- Use tables and graphs to represent and compare linear growth patterns. In particular, compare rates of change and x-and y-intercepts of different linear patterns.

Data Analysis, Statistics, and Probability

- Create and understand the formation of sample data. Identify different ways of selecting a sample.
- Select, create, and interpret data using tables, charts, **scatter plots**, **stem and leaf plots** as well as **box and whisker plots**. Differentiate between continuous and discrete data.
- Use tree diagrams, tables, organized lists, basic combinatorics, (“fundamental counting principle”), and area models to compute probabilities for simple compound events (e.g., multiple coin tosses or rolls of dice).

MATHEMATICS BENCHMARKS
ALGEBRA I

Number Sense

- Identify and use the properties of operations on real numbers, including the associative, commutative, and distributive, properties; the identity and inverse elements for addition and multiplication; the n th roots of positive real numbers for any positive integer n ; the inverse relationship between taking the n th root of and the n th power of a positive real number; and the density of the set of rational numbers in the set of real numbers.
- Simplify expressions with absolute values and positive integer exponents. Apply these simplifications to the solutions of problems.
- Find the approximate value for the solutions to problems involving square roots and cube roots without the use of a calculator.
- Use estimation to ensure that solutions to problems involving real numbers make sense.
- Use the properties of real numbers to justify the validity of equations and inequalities.

Patterns, Relations, and Algebra

- Analyze, describe and extend iterative and recursive patterns (e.g., Fibonacci Sequence).
- Solve everyday problems using systems of linear equations or inequalities. Apply algebraic and graphical methods to the solutions. Use technology when appropriate. Include mixture, rate, and work problems.
- Add, subtract, multiply, and divide polynomials and monomials.
- Solve quadratic equations by factoring, using the quadratic formula, or completing the square.
- Factor binomials, trinomials, and polynomials by utilizing factoring out the GCF, the difference of 2 squares, trinomial squares, trinomials, grouping, and canceling.
- Find linear equations that represent line either perpendicular or parallel to a given line and through a point, e.g., by using the “point-slope” form of the equation. (10.G.8)
- Solve absolute equations and inequalities, and apply these to the solution of a problem.
- Graph linear equations on the coordinate plane by utilizing the slope-intercept form of a line.
- Find a linear equation describing a line from a graph or geometric description. Explain the significance of a positive, negative, zero or undefined slope.
- Identify the domain and range of relations as well as determining whether or not a relations is a function.
- Identify dependent and independent variables of functions.
- Solve everyday problems involving linear equations and functions. Apply tablelinear, graphical, or symbolic methods to the solution.
- Use properties of real numbers to judge the validity of equations and inequalities, to prove or disprove statements, and to justify each step in a sequential argument.
- Translate between different representations of functions and relations: graphs, equations, point sets, and tabular.
- Find linear equations that represent lines, either perpendicular or parallel, to a given line and through a point (e.g., by using the point-slope form of an equation).

Continued...

Algebra 1 continued:

Data Analysis, Statistics, and Probability

- Select, create and interpret an appropriate graphical representation (e.g., scatter-plot, table, stem-and-leaf plots, circle graph, line graph, and line plot) for a set of data and use appropriate statistics (e.g., mean, median, range, and mode) to communicate information about the data. Use these to compare different sets of data.
- Describe how the sample size and the population affect the validity of predictions.

MATHEMATICS BENCHMARKS
ALGEBRA II

Number Sense

- Simplify numerical expressions with powers and roots, including fractional and negative exponents.
- Define irrational, imaginary, and complex numbers as well as the operations on them.

Patterns, Relations, and Algebra

- Identify arithmetic and geometric sequences and finite arithmetic and geometric series. Use the properties of such sequences and series to solve problems, including finding the formula for the general term and the sum, recursively and explicitly.
- Describe, complete, extend, analyze, generalize, and create a wide variety of patterns, including iterative and recursive patterns such as Pascal's Triangle.
- Demonstrate an understanding of the binomial theorem and use it in the solution of problems.
- Demonstrate an understanding of the exponential and logarithmic functions.
- Perform operations on functions, including finding inverses of functions.
- Given algebraic, numeric, or graphical representations, recognize functions as polynomial, rational, logarithmic, or exponential.
- Utilize slopes, points on a line, or intercepts to write standard linear equations.
- Solve a variety of equations and inequalities using algebraic, graphical, and numerical, methods, including the quadratic formula. Use technology where appropriate. Include polynomial, exponential, and logarithmic functions; expressions involving absolute values; and simple rational expressions.
- Use symbolic, numeric, and graphical methods to solve systems of equations and/or inequalities involving algebraic, exponential, and logarithmic expressions. Also use technology where appropriate. Describe the relationships among the methods.
- Solve everyday problems that can be modeled using polynomial, rational, exponential, logarithmic, and step functions, absolute values and square roots. Apply appropriate graphical, tabular, or symbolic methods to the solution. Include growth and decay; logistic growth; joint (e.g., $I = Prt$, $y=k(w_1 + w_2)$), and combined ($F = G(m_1M_2)/d_2$) variation.
- Describe the translations and scale changes of a given function $f(x)$ resulting from substitutions for the various parameters a, b, c , and d in $y = af(b(x + c/b)) + d$. In particular, describe the effect of such changes on polynomial, rational, exponential, and logarithmic functions.
- Solve systems of linear equations by graphing, addition or subtraction methods.
- Solve radical equations, and literal equations.
- Use matrices to solve systems of linear equations. Apply to the solution of everyday problems.
- Solve quadratic equations with real coefficients or complex roots by using a variety of methods.

Continued...

Algebra II continued:

Geometry

- Graph the circle, parabola, ellipse, or hyperbola by recognizing essential parts of each figure.
- Graph and identify the domain and range of a function.
- Define the sine, cosine, and tangent of an acute angle. Apply to the solution of problems.
- Relate geometric and algebraic representations of lines, simple curves, and conic sections. Graph the conic sections.
- Derive and apply basic trigonometric identities (e.g., $\sin^2 \theta + \cos^2 \theta = 1$, $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\sec \theta = \frac{1}{\cos \theta}$) and the laws of sines and cosines.

Data Analysis, Statistics, and Probability

- Select an appropriate graphical representation for a set of data and use appropriate statistics (e.g., quartile or percentile distribution) to communicate information about the data.
- Use combinatorics (e.g., “fundamental counting principle”, permutations, and combinations) to solve problems, in particular, to compute probabilities of compound events. Use technology as appropriate.

MATHEMATICS BENCHMARKS
GEOMETRY

- Recognize types of polygons. Identify angles and the parts of polygons. Detect symmetries of geometric figures.
- Draw and label sets of points such as line segments, rays and circles.
- Recognize types of angles formed by parallel lines cut by a transversal.
- Write simple proofs of theorems in geometric situations, such as theorems about congruent and similar figures, parallel or perpendicular lines. Distinguish between postulates and theorems. Use inductive and deductive reasoning, as well as proof by contradiction. Given a conditional statement, write its inverse, converse, and contrapositive.
- Apply properties of angles, parallel lines, arcs, radii, chords, tangents, and secants to solve problems.
- Use the 30-60-90-and 45-45-90 laws of right triangles to solve problems.
- Work effectively with coordinate geometry to find a line's slope, intercepts, and equation.
- Apply formulas for a rectangular coordinate system to prove theorems.
- Draw congruent and similar figures using a compass, straightedge, protractor, or computer software. Make conjectures about methods of construction. Justify the conjectures by logical arguments.
- Apply congruence and similarity correspondences (e.g., $\triangle ABC \cong \triangle XYZ$), and properties of the figures to find missing parts of geometric figures, and provide logical justification.
- Using rectangular coordinates, find the distance and midpoints of line segments. Apply results to the solutions of problems.
- Find linear equations of parallel and perpendicular lines to given lines and through a point.
- Solve simple triangle problems using the triangle angle sum property, and/or the Pythagorean theorem.
- Define the sine, cosine, and tangent of an acute angle. Apply to the solution of problems.
- Apply the triangle inequality and other inequalities associated with the triangles (e.g., the longest side is opposite the greatest angle) to prove theorems and solve problems.
- Demonstrate an understanding of the relationship between geometric and algebraic representations of circles.
- Draw the results, and interpret transformations on figures in the coordinate plane, (e.g., translations, reflections, rotations, scale factors, and the results of successive transformations). Apply transformations to the solution of problems.
- Demonstrate the ability to visualize solid objects and recognize their projections and cross sections.
- Use vertex-edge graphs to model and solve problems.

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Geometry continued:

- Use the notion of vectors to solve problems. Describe addition of vectors and multiplication of a vector by a scalar, both symbolically and pictorially. Use vector methods to obtain geometric results.
- Describe the effects of : approximate error in measurement, rounding on measurements and on computed values from measurements.
- Use dimensional analysis for unit conversion and to confirm that expressions and equations make sense.
- Calculate perimeter, circumference, and area of polygons and circles.
- Find the lateral area, surface area, and volumes of prisms, pyramids, spheres, cylinders, and cones.
- Find the measures of corresponding angles and sides of similar polygons.
- Relate changes in the measurement of one attribute of an object to changes in other attributes (e.g., how changing the radius or height of a cylinder affects its surface area or volume).

MATHEMATICS BENCHMARKS PRE-CALCULUS

Number Sense

- Plot complex numbers using both rectangular and polar coordinate systems.

Patterns, Relations, and Algebra

- Use mathematical induction to prove theorems and verify summation formulas, e.g.,
Verify $\sum_{k=1}^n k = \frac{n(n+1)(2n+1)}{6}$
- Relate the number of roots of a polynomial to its degree.
- Solve quadratic equations with complex coefficients.
- Determine and graph the equations of the conic sections by utilizing features such as foci, asymptotes, tangents, axis of symmetry, etc. Convert between different algebraic representations of conic sections.
- Demonstrate an understanding of the sine, cosine, tangent, cosecant, secant, and tangent functions. Relate the identity to the Pythagorean theorem.
- Explain the identity sine squared plus cosine squared equals one and its relation to the Pythagorean theorem.
- Graph trigonometric functions by utilizing periods and amplitude.
- Relate the slope of a tangent line at a specific point on a curve to the instantaneous rate of change.
- Use the remainder theorem and synthetic division to evaluate polynomials in one variable.
- Translate between geometric, algebraic, and parametric representations of curves. Apply to the solutions of problems.
- Demonstrate an understanding of the formulas for the sine and cosine of the sum or the difference of two angles. Relate the formulas to DeMoivre's theorem and use them to prove other trigonometric identities.
- Understand, predict, and interpret the effects of the parameters a,w,b, and c on the graph of $y=asin(w(x-b))+c$; similarly for the cosine and tangent. Use to model periodic processes.

Geometry

- Solve right triangles for missing sides and angles by using trigonometry.
- Use the Law of Sines and Cosines to solve for the unknown sides or angles in triangles.
- Use the notion of vectors to solve problems.
- Apply properties of angles, parallel lines, arcs, radii, chords, tangents, and secants to solve problems.

Measurement

- Describe the relationship between degrees and radian measure of angles. Use radian measures in the solutions of problems.
- Use dimensional analysis for unit conversion and to confirm that expressing equations makes sense.

Continued...

Pre-calculus continued:

Data Analysis, Statistics, and Probability

- Design surveys and apply random sampling techniques to avoid bias in the data collection.
- Apply regression results and curve fitting to make predictions from data.
- Apply uniform, normal, and binomial distributions to the solutions of problems.
- Describe a set of frequency distribution data by spread (variance and standard deviation), skewness, symmetry, number of modes, or other characteristics. Use these concepts in everyday situations.
- Compare the results of simulations with predicted probabilities.

MATHEMATICS BENCHMARKS

CALCULUS

Number Sense and Operations

- Graph functions that arise in scientific study, including lines, quadratic functions, square roots, reciprocals, and basic trigonometric functions.
- Study the absolute value function to define intervals.
- Show how derivatives are used to estimate change (differentials).
- Solve problems to find where a function is maximized or minimized.
- Determine the integration of a function known as the “antiderivative.”
- Develop, “substitution” as a tool in integration.
- Determine the absolute value of a function.
- Determine the derivatives of the inverse trigonometric functions and their related integrals.
- Determine partial fractions to help integrate rational fractions.

Patterns, Relations, and Algebra

- Derive rules for constructing the derivatives of algebraic combinations of functions (e.g., sum, product, quotient, power rules for derivatives).
- Algebraically calculate the inverse of a function.

Measurement

- Determine limits and how they are related to velocity and continuous functions.
- Using parametric equations to describe the motion of an object.
- Calculate volumes of revolution by a method known as “slicing.”
- Calculate volumes using “washers” and “cylindrical shells.”
- Determine the lengths of plane curves from two specific points.

Geometry

- Define and calculate the slopes of lines to be used to define the slopes of curves.
- Determine the net change and distance traveled, of an object.
- Find the areas between curves.
- Graph conic sections by determining center points, vertices, foci, and their formulas.
- Calculate the area of a region that has curved boundaries.

Data Analysis, Probability and Statistics

- Determine where a graph increases and decreases by using the 1st derivative.
- Determine the concavity of a graph by using the 2nd derivative.
- Use derivatives to approximate a function at a point (linearization).
- Use methods of integration to determine the solving of integration by parts and the solving of tabular integration.